Characteristics of a Standards-Based Mathematics Classroom

To support efforts to improve the teaching and learning of mathematics across the state, in the summer of 2005 the Massachusetts Department of Elementary and Secondary Education (ESE) launched the Comprehensive School Reform Math Initiative. Each district participating in this initiative received grant funding to support a full-time staff position dedicated to leadership of the district's math initiative. These leaders, in cooperation with ESE, come together regularly as the Math Support Specialist (MSS) Network to share ideas, resources, and strategies related to K-8 mathematics education reform.

ESE's Mathematics Targeted Assistance team and the MSS Network participants have developed a shared vision of standards-based mathematics teaching and learning to guide this new collaboration. Based on this vision, we have articulated the characteristics of an effective standards-based mathematics classroom and their corresponding indicators to serve as a reference for instructional planning and observation. This document represents the present state of this work. It is intended to support activities that advance standards-based educational practice, including formal study, dialogue and discussion, classroom observations, and other professional development activities.

A Shared Vision of Standards-Based Mathematics Teaching and Learning

Standards-based mathematics teaching and learning is a cooperative effort by teachers and students to actively engage in purposeful learning experiences that stimulate curiosity, enjoyment, and deep understanding of the mathematical concepts outlined in the Massachusetts Mathematics Curriculum Framework. Teachers and students are knowledgeable about learning objectives, and have ownership of and are accountable for learning outcomes.

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The original 2006 document developed in collaboration with ESE and the Math Support Specialists network has been modified to increase its sensitivity for use in classrooms providing math instruction to English Language Learners. In this draft version, additions to the document are in a bold italic font. While most of the original (and many additional) indicators represent good practice for all students, many are imperative for effective teaching and learning for ELL students. The indicators are intended to be suggestive, not exhaustive, identifying what effective implementation of the associated characteristic might look like in classroom practice. It is expected that this document will provide helpful, practical guidance for those who teach and those who support instruction for English Language Learners.

I. Student Learning Standards

Characteristic 1.1 **Learning Standards Are Evident**
Characteristic: The mathematics learning standards being addressed in the lesson are evident and clear to the students.
Indicators: • ***Teacher introduces standards using language and/or representations appropriate to the age and/or English language proficiency of students.*** • Standards are clearly visible, and specific verbal reference is made to the standards that students are expected to understand. • Connections are explicitly made with learning standards presented in previous and subsequent lessons (i.e., this lesson does not occur in isolation).
Characteristic 1.2 **Exemplars**
Characteristic: Exemplars demonstrate expectations of student achievement.
Indicators: • Students have concrete examples/models of high quality products (teacher-generated, student-generated or both) that represent mastery of the learning standard(s). • Students have descriptions, written or oral, in age-appropriate language, of what constitutes a high quality product (e.g., a descriptive rubric).

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II. Organization of the Lesson

Characteristic 2.1: **Lesson Well-Planned and Organized**
Characteristic: The lesson is well planned and organized. The objectives of the lesson are clearly stated and connected with the learning standards of the larger unit of which it is a part. The lesson develops in a clear, logical manner.
Indicators: • ***Content and Language objectives are explicit.*** • The plan for the day appears in writing, in age-appropriate language (e.g., an agenda). • There is a logical flow to the lesson, ***with access for Limited English Proficient students to participate meaningfully in learning activities and fulfill assigned tasks.*** • Lesson objectives are communicated verbally, ***visually*** and/or in writing in age-appropriate language. • All components of the lesson (learning activities, homework, assessment, etc.) contribute to the lesson objectives and to mastery of the standard(s). • ***The lesson builds upon everyday language to introduce formal academic English.*** • ***Instruction is delivered using language (vocabulary, sentence structures) that is comprehensible to students, building toward formal academic English.***
Characteristic 2.2: **Time Used Effectively and Purposefully**
Characteristic: Time is used efficiently and purposefully.
• Students begin doing math work soon after class begins. • Students follow classroom routines well enough that minimal time is spent on receiving directions. • Minimal time is spent on organizational details (attendance, distribution of supplies, etc.). • Time spent on homework supports the lesson. • The majority of class time is spent developing new knowledge. • More student time is spent actively engaging in mathematics than passively receiving instruction about mathematics. • Sufficient time is allotted to conclude the lesson in a meaningful, appropriate way.
Characteristic 2.3: **Multiple Grouping Strategies**
Characteristic: Multiple grouping strategies are used to achieve the learning that is the object of the lesson (e.g., individual, small groups, whole class, teacher-student).

Indicators:
• Possible grouping configurations *(expect to observe two or three):*
  o Part of the lesson involves the entire class.
  o Part of the lesson involves small groups.
  o Part of the lesson involves students working in pairs.
  o Part of the lesson involves students working individually.
• Each configuration and composition of groups is appropriate for the task to be accomplished.
  o ***At times, Limited English Proficient students may use primary language to support access to math (e.g., for clarification of content, to allow increased opportunities for interaction), as a bridge to connecting conceptual understanding in academic English.***
• There are clear guidelines and expectations for group work.
• All groups are supported as they extend their ability to learn and do mathematics.

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III. Classroom Environment

Characteristic 3.1: **Safe Environment**
Characteristic: It is clear that the students appear to feel safe and are willing to take risks.
Indicators: • Positive, respectful relationships are evident within the classroom (teacher – student, student – student). • ***Established routines make expectations clear to students.*** • Expectations about supportive learning relationships are explicit. • All communication within the classroom is respectful and appropriate. • The dynamics of the classroom support risk-taking in mathematical discourse, in which students question and contribute and collaborate throughout the lesson.
Characteristic 3.2: **Physical Organization**
Characteristic: The appearance and physical organization of the classroom contribute to a positive learning environment.
Indicators: • Student work is displayed demonstrating writing and problem solving related to the mathematics standards. • The space is physically arranged as an efficient, functional environment. • The desk/table arrangement allows for teacher mobility/accessibility. • The desk/table arrangement allows for a variety of activities. • The climate of the room (temperature, air quality, light, cleanliness) is conducive to education.

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IV. Student Learning

Characteristic 4.1: **Student Engagement**
Characteristic: Students are actively engaged in all aspects of the lesson. Behavior is appropriate for the lesson/activities.
Indicators: • Verbal and non-verbal cues indicate student engagement (e.g., questions, responses, eye contact, attentiveness, posture). • Students have and utilize required materials (e.g., textbook, homework, pencil). • Students follow directions and accomplish all assigned tasks. • Inappropriate behavior is reasonably addressed consistently. • Students are focused on mathematics throughout the lesson. • Conversation is on-task and appropriate. • Students demonstrate respect for property and materials. • ***Teacher uses awareness of cultural differences to address issues that may preclude students from interacting in ways typically considered productive (e.g., social norms regarding eye contact, questioning teachers) with the goal of teaching students norms of an American classroom.***
Characteristic 4.2: **Various Ways of Learning**
Characteristic: Students are engaged in understanding and learning mathematics in various ways that include skill building, conceptual understanding, applying multiple problem-solving strategies, and real-world applications.
Indicators *(expect to see two or three)*: • Students learn and practice mathematical skills, facts, procedures and algorithms. • Students explore and discuss mathematical concepts. • Students use problem-solving strategies. • Students learn mathematics in the context of real-world problems and applications.
Characteristic 4.3: **Students Examine Thinking and Support Reasoning**
Characteristic: Students consciously examine their thinking by questioning their understanding of the mathematics presented. Students support and defend their reasoning with data while using appropriate mathematical language.
Indicators: • ***Student interactions in a variety of contexts support the development of both mathematical (formal academic) and everyday language.*** • Students support their reasoning with data and evidence. • Students apply algorithms purposefully in problem-solving situations. • Students develop multiple problem-solving strategies. • Students use mathematical language that includes vocabulary related to the lesson. • Students demonstrate and articulate their mathematical reasoning. • Student questions and comments indicate mathematical reflection, understanding and development.

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V. Teaching

Characteristic 5.1: **Content Knowledge**
Characteristic: Depth of content knowledge is evident throughout the presentation of the lesson. Mathematical concepts are presented accurately.
Indicators: • All mathematics explained and demonstrated throughout the lesson is sound and accurate. • Mathematical concepts and ideas are explained in multiple ways to enable student understanding. • Mathematical connections are made across ideas and strands. • Mathematics is presented as a system of ideas, concepts and understandings, not simply as unrelated procedures, facts and algorithms.
Characteristic 5.2: **Probing Questions**
Characteristic: Through the use of probing questions and student responses, decisions are made about what direction to take, what to emphasize, and what to extend in order to build students' mathematical understanding.
Indicators: • Questions require more than one-word responses. • ***Teacher allows sufficient time for students to process input and formulate their responses.*** • The level of student understanding, evidenced by student responses, directs how the discussion moves. • Questions scaffold progression to higher levels of mathematical thinking. • ***Probing questions are used and student work is examined to assess student understanding with respect to language and math content.***
Characteristic 5.3: **Students' Prior Knowledge**
Characteristic: Students' prior knowledge is incorporated as new mathematics concepts are introduced. When students raise comments, questions, and/or concerns, their perspectives are acknowledged and either redirected or affirmed, linking existing knowledge to new knowledge gained within the lesson.
Indicators: • The lesson requires students to draw upon their existing knowledge of mathematics. • Students draw on their existing knowledge and their experience of the world around them to inform their learning. • Students are given time and opportunity to express their understandings and ideas, which are discussed respectfully and used to scaffold learning. • Connections are explicitly made between students' prior mathematical knowledge and the new ideas being introduced in the lesson, ***including culturally related conventions and algorithms (e.g., use of comma vs. decimal point).*** • ***Cognates are used to provide students access to concepts (e.g., equal-igual, to estimate-estimar, capacity-la capacidad).***

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Characteristic 5.4: **Student Misconceptions**
Characteristic: Student misconceptions are anticipated/identified and addressed.
Indicators: • Student misconceptions are anticipated and addressed***, including language-based misconceptions. (E.g., In a mathematical context, the question: "Does this work for <u>any</u> triangle?" actually means: "Does this work for <u>all</u> triangles?")*** • As misconceptions are identified, students are respectfully redirected to develop accurate mathematical thinking and understanding. • Students are provided opportunities to identify and correct their own misconceptions through mathematical exploration and discussion. • Students respectfully correct each other's misconceptions.
Characteristic 5.5: **Multiple Forms of Representation**
Characteristic: Classroom strategies incorporate multiple forms of representation (e.g., pictures, words, symbols, diagrams, tables, graphs).
Indicators: • Mathematical content is expressed in multiple ways (e.g., pictures, words, symbols, diagrams, tables, graphs). • Opportunities are provided for students to understand that various representations may all express the same mathematical concept. • Teacher provides opportunities for students to use multiple representations as they develop and explain mathematical ideas. • ***Key words are introduced and reinforced throughout the lesson.***

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VI. Technology

Characteristic 6.1: **Instructional Tools**
Characteristic: Appropriate tools for learning are provided (e.g., measuring instruments, manipulatives, calculators, computers). All necessary resources for the lesson are easily accessible. Instruction and support are provided for use of tools.
Indicators: • All tools appropriate for the lesson are available in sufficient quantity to students (e.g., measuring instruments, manipulatives, calculators and computers). • Learning tools are easily accessible and functional. • Use of manipulatives and technology are connected to the lesson objectives (i.e., the technology is not used for its own sake). • Students are given sufficient instruction and support regarding the use of learning tools.

VII. Equity

Characteristic 7.1: **High Expectations for All Students**
Characteristic: There are high learning expectations for all students. All students participate, and their ideas are valued. The belief is evident among all in the classroom that effort, not innate ability, is the key to significant mathematical learning.
Indicators: • All students are expected to become proficient in the standard(s) addressed in the lesson.   o ***Amplify and enrich the learning experience without simplifying the language of the classroom, to give students more opportunities to learn the concepts involved.*** • Students with special needs are supported as appropriate (e.g., as outlined in IEP). • All students, regardless of current knowledge, are provided entry into the lesson enabling mathematical learning. • Wait-time is used effectively to allow all students meaningful participation. • Students are provided with opportunities to experience achievement through the application of effective effort.
Characteristic 7.2: **Variety of Learning Experiences**
Characteristic: Various learning experiences are provided that are appropriate for the range of learners in the classroom (i.e., differentiation by content, process, and/or product).
Indicators: • Students engage in appropriate activities in terms of complexity and pacing for their current level of understanding and skill, but which challenge them to move forward. • Students are given opportunities to relate their personal and academic interests to their learning of mathematics. • Mathematics is presented to students in ways that are responsive to individual learning styles and ways of knowing. • ***Multiple representations are used to provide access to content for English Language Learners (e.g., communication through movement, gestures, visuals, music).***

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VIII. Assessment

Characteristic 8.1: **Multiple Types of Ongoing Assessment**
Characteristic: There is evidence of multiple types (e.g., group/individual presentations, written reflections, tests) of diagnostic and ongoing formative assessment.

Indicators:
• Data from continuous assessment is used to inform instruction.
• Understanding is assessed through:
  o Student responses to questions
  o Group interactions
  o Student work
  o Student/group presentations
  o Journals/written-reflections
  o Student projects
  o Tests and quizzes
• ***Students are provided opportunities to express understanding of mathematics in ways appropriate for their language proficiency (e.g., visual, gestures).***

Characteristic 8.2: **Student Ownership of Learning**
Characteristic: Students are engaged in and responsible for their own learning, examining their results with directive feedback that enables revision and improvement.
Indicators: • Students take initiative to develop and further their own mathematical learning. • Students receive information (from teacher or other students) that helps them understand their level of mastery regarding the standard(s). • Students receive direct feedback to explicitly guide continuous progress toward mastery of the standard(s). • Students are given opportunities to revise their work.