Article #1

TLL Library

"Transforming Novice Problem Solvers Into Experts,"
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The first of three *Teach Talk* *columns to focus on the implications of research into learning for actual classroom practice.*

It's a common refrain that came up again recently during a conversation among several faculty members after a seminar on new educational technologies. The discussion had winded its way around to the intellectual strengths and weaknesses of the students, and the question popped up, as it often does around this subject: Why can't students be better problem solvers? Professor Heidi Nepf from Civil and Environmental Engineering summed up the faculty's sense of frustration particularly well. "I can give my students a set of problems that all follow a certain model, and they'll do fine," she said. "The minute I throw in a novel condition or create a problem that doesn't look like something they've seen before, they're lost." Then she turned to me and asked, "How come?"

I don't think anyone would argue that the problem is a complex one. It is connected to such factors as the kind of high school education our students received, their own proclivities, and their stage of intellectual maturity. But I'd like to suggest that at least part of the answer lies in the fact that too often we don't explicitly teach students the process of problem solving. We expect that as they listen to us in lecture or watch us in recitation they will somehow absorb the skills they need to make the jump from using "plug 'n' chug" to employing more sophisticated problem solving strategies. But as Donald Woods, professor emeritus of chemical engineering at McMaster University and a leading developer of problem-based learning curricula, writes, "In a four-year engineering program, students observed professors working more than 1,000 sample problems on the board, solved more than 3,000 assignments for homework, worked problems on the board themselves, and observed faculty demonstrate the process of creating an acceptable internal representation about fifteen times. Yet despite all this activity, they showed negligible improvement in problem-solving skills . . . .." (Donald Woods, "How Might I Teach Problem Solving," in J. E. Stice, ed., Developing Critical Thinking and Problem-Solving Abilities. New Directions for Teaching and Learning, no. 30, 1987, pp.58-59) Yet I don't think instructors should be blamed: My guess is that if a representative sample of MIT faculty were asked to describe how they go about solving problems, they wouldn't be able to. In that regard, they wouldn't be any different from most experts who have so internalized their problem solving abilities that these skills have become transparent to them.

Happily, thanks to the work of cognitive psychologists, educators, and researchers in artificial intelligence, who have been studying problem solving for at least the last 30 years, we do know something about how skilled problem solvers recognize, approach, and ultimately solve problems. Much of this research has revolved around examining what distinguishes expert problem solvers from novices. Educators have then gone a step further to develop methods that can be used both inside and outside of the classroom to strengthen the novice's problem solving skills.

In this *Teach Talk* I'd like to focus on the expert/novice dichotomy, because I believe it contains an especially rich lode of information regarding the skills our students need to develop. In fact, this column is the first of three *Teach Talks* that will be devoted to describing recent research in learning in higher education. (The next two columns will deal with the theories of constructivism and situated learning.) Each column is designed to inform readers on how this research can be applied to improving actual classroom practice, for this knowledge has direct implications for structuring the MIT educational experience.

The Components of Problem Solving

The most useful definition I have found for problem solving begins by conceptualizing a continuum that runs from "learning" to "problem solving" to "creativity." In this schema, learning refers to the students' ability to demonstrate they have internalized the material to which they have been exposed by displaying it in a context similar to that in which they were taught. "Transfer of learning" is demonstrated when the situation is somewhat different from the original one. If, however, the transfer situation is substantially different from the original, or if students meet some barrier or difficulty in using the learning, then they are faced with problem solving. (This is the situation to which Professor Nepf referred.) Creativity is at the far end of the continuum where the situation is so vastly different that what has been learned is transferred to a totally new context.

Several scholars, including Donald Woods, have sought to break down the process of problem solving into its component parts. Woods' six-step plan, which he credits as an extension of the plan devised by György Polya in his classic book How to Solve It, directs problem solvers to: read about the situation; define the given situation or problem; define the "real" problem and create a "representation" of it (more on this below); plan; do it; and check, look back, and implement. Woods further decomposes each step into smaller parts. For example, "defining the situation" (step two) is rooted in analysis, which consists of reasoning, classifying, identifying series and/or relationships, creating analogies, and checking for consistency. While there may be disagreement about the exact nature or order of the steps in the problem solving process, the underlying point remains valid: Problem solving can be dissected into a set of skills that students can be exposed to along with course content. One cannot substitute for another. (Interestingly, attempts to teach problem solving as a separate course have not been as successful as when problem-solving skills are interwoven into a "content" course. Giving students problems from the "real world" and using those problems as the basis for teaching problem solving is particularly effective. In fact, Woods maintains that the types of problems students are typically given in science and engineering classes are not appropriate at all for teaching problem-solving skills.)

Finally, while we are likely to think of problem solving as a cognitive capability, a number of researchers have also looked at the role of attitudes, values, beliefs, and emotions in successful problem solving. (Actually, the research of neurologist Antonio Damasio suggests that emotion and cognition should not be viewed as separate activities in the brain at all; rather, they work in concert.) We know, for example, that if students believe they are incapable of solving a certain kind of problem, they are likely to be unable to do it. De Bellis and Goldin have examined the "influence of values, i.e., one's psychological sense of what is right or justified, on problem solving," report Annie and John Selden in "What Does It Take to Be an Expert Problem Solver?" The Seldens go on to write, "For example, some students may feel they 'should' follow established procedures, whereas others may value originality and self-assertiveness." (MAA Online, 8/30/97, p. 4) Other students who feel they should know the answer to a problem may become easily frustrated, which can "lead them to guess or use plausible, but inappropriate, procedures," the Seldens write. (MAA Online, 8/30/97, p. 4)

Good problem solvers are more often than not intrinsically motivated by curiosity, challenge, and fantasy. (Joanne Gainen Kurfiss, "Critical Thinking: Theory, Research, Practice, and Possibilities," ASHE-ERIC Higher Education Report No. 2, 1988, p. 47) Good problem solvers are not daunted by the unknown, but are challenged by it. They may experience frustration in their work, but it doesn't defeat them; instead, it spurs them on. What else differentiates the experts from the novices?

What Do the Experts Do?

There are a number of characteristics that differentiate the expert from the novice problem solver. But at the heart of the matter is that experts think about, consider, and examine the problem as a whole before beginning to work on a solution. They classify a problem according to its underlying principles, deciding to what class of problem it belongs. They engage in a planning stage before even attempting a solution. Novices jump right in.

In a classic 1978 study comparing individuals who were expert at solving problems in physics with novices, Simon and Simon found that experts use a "working forward" method, looking at the givens of the problem first and moving from the statement of the problem to a physical representation of it. Only after they do this analysis, identifying likely ways to reach an answer, do they employ equations. Then they call upon successive layers of equations, first using ones that can be solved with the givens in the problem. They also add information that will help them solve the problem from their own reservoir of learning. The experts' use of equations, in other words, is guided "by the planning already done." (D. P. Simon and H.A. Simon, "Individual Differences in Solving Physics Problems," in R. S. Siegler, ed., Children's Thinking: What Develops? 1978, as reported in Larkin, Heller, and Greeno, "Instructional Implications of Research on Problem

Solving," New Directions for Teaching and Learning, 2, 1980, pp. 55-57)

Novices, on the other hand, use a "working backward" strategy trying to determine what procedure will get them to an answer. They tend to take more "piecemeal approaches" (Larkin, Heller, and Greeno, p. 59), working by trial and error. They memorize, then try to apply equations independent of context or any relationship to the inherent characteristics of the problem. Especially problematic is that they try to translate the problem directly into a mathematical representation, using a means-ends analysis. Or as one writer characterized it "[they] . . . select a 'first impression solution.'" "In effect," write Larkin, Heller, and Greeno, "experts understand problem situations better than novices." (p. 59)

The good news is that when studies compared successful students with those having difficulty solving problems, the former looked much like the effective problem solvers of the Simon and Simon study. Successful students are able to apply specific pieces of knowledge to help answer the problem. Unsuccessful students can't relate what they have learned to the question if the question is asked in a form that is different from the one they have seen. (Greenfield, p. 15) Successful students work more actively; unsuccessful students more passively. Successful students are careful and systematic. Unsuccessful students leap into a problem with at best a haphazard plan, move without direction, and are unable to focus on any particular starting point. Their knowledge base has no hierarchical organization to it, and they are easily distracted by some difficulty or something irrelevant. On the other hand, like their professional counterparts, successful students begin with a plan, modifying it as needed. They carefully develop and organize their knowledge base, structuring it around fundamental principles and abstractions. (Greenfield, p. 15)

If we accept the premise that good problem solvers are made and not born (allowing, of course, for differences in innate capabilities), and that we have a responsibility to instruct in this area as well as in content, the simple question is, how? In other words, what are the implications of this research for what happens in our classrooms?

Teaching Problem Solving

I'd like to reiterate what I wrote earlier: The process of problem solving has to be taught explicitly if we want to raise the general level of students' problem-solving abilities. Although many students will eventually internalize the habits of good problem solving, this can occur earlier for more students if the necessary skills are described, modeled, and practiced, and if the instructor provides students with feedback on their behavior. As with many skills, learning happens when a discussion of best practices are combined with opportunities for learners to try their hands at the skill, and are told both what they are doing correctly and how to improve.

Greenfield suggests six things instructors can do to teach problem solving. They should:

- model problem solving (making an occasional error or going down a blind alley is good!) so that students see the process is not straightforward or linear;
- demonstrate there is more than one way to solve a problem, so that students don't look for the one right way;
- redescribe the problem in qualitative terms and apply relevant underlying principles;
- help students create a plan for the solution, estimating the range in which the answer might lie;
- show how to break the problem down into manageable parts, identifying and clarifying key concepts, drawing a diagram, translating the problem into a simpler form;
- help identify and isolate factors that might lead to wrong solutions and develop strategies to counteract these problems. (p. 19)

The author also suggests using the "think aloud" process first developed by Jack Lochhead and Arthur Wimbey in the early 1980s. In this instructional method, two students work together to solve a series of short problems. One student becomes the problem solver, and he/she reports out loud everything that is going on in his/her head as he/she attacks the problem. The other student is the listener whose "primary objective," write Lochhead and Wimbey, "is to understand in detail every step and every diversion or error made by the problem solver." The listener can also use a checklist that the authors have developed to help him/her notice errors in the problem solver's reasoning process. ("Teaching Analytical Reasoning through Thinking Aloud Pair Problem Solving," in Stice, p.75) After the first student solves his/her problem, the two students switch roles and work on another problem. There are obviously a number of benefits to this method: students call direct attention to the process they are using and reflect on it; the process is monitored and can be called into question by another; and students practice working with others as they will be doing in the professional world.

Some educators say that what is needed is a "cognitive apprenticeship" approach to instruction. The elements of such a pedagogical method would consist of modeling, coaching, scaffolding (i.e., providing expert guidance a practice working with others as they will be doing in the professional world. Some educators say that what is needed is a "cognitive apprenticeship" approach to instruction. The elements of such a pedagogical method would consist of modeling, coaching, scaffolding (i.e., providing expert guidance at the beginning of the process and then removing it), articulating, reflecting, and exploring. (Kurfiss, p. 45) This is a very different model from the one in which the instructor does the problem solving for the class, but doesn't reveal the "secrets" of his/her success. If we want students to be better problem solvers, we have to be like magicians who are willing to show our audience how we do our sleight of hand. If we want students to be better problem solvers, we need to be better teachers of the process for solving those problems.

Article #2

HELPING STUDENTS MAKE THE TRANSITION FROM NOVICE TO EXPERT PROBLEM-SOLVERS

Michael Prince[1], Brian Hoyt[2]

Abstract *— Engineers, by definition, need to be good problem solvers. This paper discusses a model for building on a traditional engineering curriculum to systematically develop students' problem solving skills. The curriculum structure consists of required courses that emphasize problem solving at distinct levels. The courses are broken down into introductory, intermediate and advanced problem solving courses. The type of problems utilized in each course differentiates the courses. The problems posed are qualitatively different, not simply harder, thus requiring the students to engage different skill sets for resolution. As a result, the courses develop different problem solving abilities.*

The model for teaching problem solving has been developed through Project Catalyst, which is an NSF funded initiative to improve undergraduate engineering education. This paper presents the details of the proposed model, discusses educational modules that have been developed to aid instructors introducing problem solving in their courses and provides some initial assessment of the results to date.

Index Terms —Curriculum Design, Problem Based Learning, Teaching Problem Solving.

INTRODUCTION

Engineers, by definition, need to be good problem solvers. In fact, the Accreditation Board for Engineering and Technology (ABET) now requires that all engineering programs demonstrate that students have the ability to "identify, formulate and solve engineering problems". Few engineering faculty would disagree with the importance of this criterion. However, the traditional undergraduate engineering curriculum is not designed to systematically develop relevant problem solving skills. Consider, for example, that the *bulk* of the curriculum emphasizes facts, formulas and low level textbook exercises. In fact, an analysis of one four- year engineering program found that approximately 80% of problems assigned to students required only low-level thinking skills [8]. The authors classified problems using Bloom's taxonomy, and concluded that most problems did not require any analysis, synthesis or evaluation. In addition, a traditional engineering program reserves most of the higher level thinking, such as design, until the senior year. And finally, a traditional program relies on constant repetition of textbook problems to develop problem solving skills but would typically not include any formal training in problem solving methodologies.

What is the problem with this approach? Most of us have gone through programs like this and may have taught this way for years. However, there is reason to think that we can do better. At Bucknell University, a group of engineering faculty involved in an NSF funded initiative to re-envision engineering education [3] has developed a different model to teach problem solving based on several arguments, described below:

How We Learn Skills

We all acquire skills in one way and one way only, though practice and feedback. Students learn how to identify, formulate and solve engineering problems by identifying, formulating and solving engineering problems and then getting feedback to learn from their experience. In a proper educational environment, guidance as well as feedback would be provided.

How does this relate to a traditional engineering program with the heavy emphasis on textbook exercises? It is clear that one of the critical flaws of relying heavily on textbook problems is that they do not generally require relevant problem solving skills. The textbook authors have already identified and formulated the problem, which is now an exercise that typically requires only application of material from that chapter to solve. That is not "problem solving", in any real sense and would not satisfy the accreditation criterion on problem solving or prepare students for industrial practice.

The Importance of Context

Even recognizing the limitations of traditional textbook problems, some may argue that textbook problems build the foundation for more relevant things. There's probably some truth in that. However, textbook problems are artificial and generally lack a relevant context, or at least one that is genuinely relevant to the student. For that reason alone, textbook exercises aren't ideal teaching tools. Perhaps more importantly, there is evidence to suggest that students who only solve textbook problems are not likely to be able to apply the concepts to real problems [2]. In response to some of these concerns, many of the authors have adopted the use of problem-based learning in their courses and have structured the classes so that relevant and realistic problems drive most of the learning that occurs [7].

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Principles of Good Instructional Design

One of the principles of good instructional design is to develop student skills and responsibilities in a gradual way so that students make the transition from novice to expert problem solver easily over time. Students should be introduced to relevant problem solving early in the curriculum and gradually encouraged and trained to adopt appropriate problem solving skills. This assumption about instructional design underlies the tiered curriculum structure described in this article and elsewhere [4].

DEVELOPING PROBLEM SOLVING SKILLS: A STAGED CURRICULUM MODEL

Based on these ideas, the authors have developed a tiered curriculum model to develop problem solving skills throughout the engineering curriculum and have begun to implement the model at Bucknell University. The curricular structure to promote problem-solving skills consists of core courses phased throughout the curriculum that emphasize problem solving at distinct levels. The courses are broken down into introduction problem solving courses (P1), intermediate problem solving courses (P2) and advanced problem solving courses (P3). The type of problems utilized in each course differentiates the courses. The problems posed are qualitatively different, not simply more difficult, thus requiring the students to engage different skill sets for resolution. As a result of the distinct problem types used, the courses develop different problem solving abilities. Since these courses are staged throughout the 4-year curriculum, students gradually receive practice and instruction in a broad range of problem solving skills. As a result, students are gradually weaned away from textbook problems and develop more practical problem solving abilities.

Table 1 provides a definition and example of the type of problem encountered at each level of problem solving in the curriculum. The table also identifies where courses tend to fall in the four- year curriculum and maps learning outcomes associated with each course to Bloom's taxonomy. One can see that there is a steady progression in the range of problem solving skills required and in the level of Bloom's taxonomy as students move through the levels of problem-solving in the undergraduate curriculum.

While there is not a complete separation of problem type used by course designation, a designated course will *emphasize* problems of a certain level. Therefore, while an intermediate problem-solving course may contain some P1 and P3 type problems, the major emphasis will be on vaguely defined problems requiring significant problem definition on the part of the student. A more detailed description of each level of problem solving is described below.

INTRODUCTORY PROBLEM SOLVING COURSES

Introductory problem solving or P1 courses emphasize well-defined problems having unique solutions and often unique solution methodologies. These are the types of exercises that are frequently found at the end of textbook chapters. Many of these problems rely on "problem recognition" and applying known algorithms. In introductory problem solving courses, students must develop the knowledge base to recognize the problem, choose an appropriate algorithm and execute it. For example, students in a course on heat transfer might be asked to calculate the heat flux through a wall, given the wall materials, thickness and temperatures of each surface.

This type of problem solving happens in many classes. While routine, it develops skills that are prerequisites for more advanced problems. In addition to providing the technical knowledge base necessary for engineering practice, introductory problem solving courses can be used to develop a number of general problem-solving skills. Specific learning outcomes associated with introductory problem solving courses include the ability to:

- recognize routine engineering problems and choose appropriate solution algorithms.
- map out a solution plan.
- obtain relevant information necessary to solve the problem.
- make and evaluate appropriate assumptions.
- draw appropriate conclusions.

Some of these skills, especially the ability to recognize a problem and plan a solution strategy, are elements of several published problem-solving methodologies. Therefore, instructors might think about introducing students to a formal problem solving methodology in introductory or subsequent courses emphasizing problem solving. We have introduced students to the well-known methodology of Donald Woods [6] because of its wide recognition and acceptance in engineering education. The specific methodology adopted is not central to the curriculum structure proposed in this article, though Woods makes an articulate argument for his approach and provides a good overview of the literature for interested readers.

INTERMEDIATE PROBLEM SOLVING COURSES

Intermediate problem solving or P2 courses utilize problems that are more realistic in that they are vaguely defined. The significant difference from P1 courses is that intermediate problem solving courses emphasize problem definition in a way that is not present in introductory problem solving courses. This is accomplished by phrasing the problem in such a way that there is some ambiguity and uncertainty. A common approach used in these courses is to embed the problem in a scenario that one might encounter if one were a consultant and just hired by an organization to correct a problem.

Using the example in Table 1, students in a heat transfer course might be asked to assume the role of an engineering consultant brought in to analyze why the heating system does not maintain a comfortable room temperature. A common reason might be that the system is not adequately sized to handle the heat loss from the room, which students can determine by examining the specifications of the heating system and the relevant room characteristics. Students, however, must examine the problem and do the required analyses to determine the problem. Only then can they make a rational recommendation to address the problem.

The use of ill-defined problems develops critical problem solving skills that our students need. However, this is not necessarily design, nor does it require a great deal of creativity or synthesis. While having upped the ante, so to speak, by requiring significant and practical problem solving skills, the problems differ from those found in traditional design courses in that no significant amount of real design is necessary. However, there is a critical difference from P1 courses in that *students* must put the problem into a solvable form. Only then can students apply appropriate algorithms to complete any necessary calculations to solve the problem.

As with introductory problem solving courses, there are specific learning outcomes associated with intermediate problem solving courses that are independent of technical content. The generic learning outcomes associated with intermediate problem solving courses are:

- Those from P1 courses, which are foundational.
- The ability to define a problem.
- The ability to assess that the solution developed adequately addresses the given problem.

ADVANCED PROBLEM SOLVING COURSES

Advanced problem solving or P3 courses emphasize problems that require significant elements of creativity. These might be the types of problems found in senior design courses. Here, design is described as ill-defined problems (poorly defined problem statements, goals or both) with multiple solutions and solution methodologies possible. In essence, the magnitude of the ambiguity changes from intermediate courses. The problems become one of scale and scope. The students are asked to start at the beginning and to build something rather than fix something. If the instructor is embedding the problem in the context of a consulting problem, the student as consultant might be asked to design a plant of some sort—which would be different from the type of ill-defined problem encountered in an intermediate problem-solving course. Advanced problems allow for more creativity and for more errors.

The specific learning outcomes associated with advanced problem solving courses include:

- All of the skills developed in P1 and P2 courses.
- Ability to generate creative solutions to address the real problem.
- Ability to evaluate and choose among multiple possible solutions.

ASSESSMENT OF PROBLEM SOLVING

Assessment of results of the curriculum structure to develop problem-solving skills is preliminary at this point. We are still in the process of developing appropriate modules and instructor materials to develop problem-solving skills. We are also still in the process of fully integrating the staged approach for problem solving into the curriculum. However, we have systematically surveyed both faculty and students involved in Project Catalyst on the effectiveness of the courses for developing problem-solving skills.

Because the Chemical Engineering program has achieved the highest level of curriculum integration at this point, the survey results are shown from chemical engineering courses in the sophomore, junior and senior years. Those results are shown in Table 2. While survey data are only one measure of the effectiveness in achieving. learning outcomes, there is some evidence to suggest that survey data correlate reasonably well with other objective measures. For example, Pike [4] found that self-reported measures of educational gains were as valid as objective measures to the extent that the self-report measures reflected the content of the learning outcomes under consideration.

TABLE 1.
STAGED LEVELS OF PROBLEM SOLVING

Course Level	Definition	Example	Bloom's Taxonomy
P1: Introductory Problem Solving	Recognition and application of routine algorithms	Calculate the heat flux through a wall of known composition	Knowledge, Comprehension, and Application
P2: Intermediate Problem Solving	Solution of poorly-defined problems requiring students to reformulate problem into a solvable form before applying algorithms.	Determine why a room's heating system does not maintain a comfortable temperature	Analysis
P3: Advanced Problem Solving	Solution of open-ended, vaguely-defined problems requiring significant creativity. Comparing alternative design solutions.	Design a new heating system for a room that meets size and cost constraints.	Synthesis and Evaluation

TABLE 2
Student and Faculty SURVEY DATA ON PROBLEM SOLVING

Questions for Spring of 2001	Student Response[1]	Faculty Response
This course was effective in developing students' abilities to analyze and evaluate problems beyond the simple recall of facts.	6.61†	6.33
This course satisfactorily developed students' abilities to integrate course material to solve open-ended problems.	6.45	5.67
The collaborative learning format of the course was more effective in developing problem-solving skills than a traditional lecture based approach	6.59	6.67
The collaborative learning format of the course was more effective in developing critical thinking than a traditional lecture based approach	6.44	6.67
Questions for Fall of 2001	**Student Response[2]**	**Faculty Response**
This course was effective in requiring students to use knowledge gained previously from other courses in the curriculum.	6.08†	6.29
This course was effective in developing students' abilities to solve problems that are vaguely defined or that have more than one acceptable solution.	6.27	6.14
This course was more effective than a lecture-based format‡ for requiring students to use knowledge gained previously from other courses in the curriculum.	5.84	6.57
This course was more effective than a lecture-based format‡ for developing students' abilities to solve problems that are vaguely defined or that have more than one acceptable solution.	6.02	7.00

1. Student data taken from 3-targeted courses in the chemical engineering curriculum. Fluid Mechanics in the sophomore year, Unit Operations laboratory in the junior year and Advanced Design in the senior year. Each course stressed elements of teamwork at different levels.
2. Student data taken from 4-targeted courses in the chemical engineering curriculum. Chemical Engineering Principles in the sophomore year, Heat and Mass Transfer and Equilibrium St age Processes in the junior year, and Design in the senior year. Each course stressed elements of teamwork at different levels.

† All responses on a 7-point scale: 7-highly agree, 6-moderately agree, 5-slightly agree, 4-neutral, 3-slightly disagree, 2-moderately disagree, 1-highly disagree.

‡ For purposes of this survey, a "lecture-based" format was defined to be one where the professor sets the agenda and lectures for the majority of the time in class.

The results show a high degree of consistency between students and faculty and from course to course. Both students and faculty moderately to strongly agree that the targeted courses were effective for developing a range of problem solving skills. In addition, both students and faculty moderately to strongly agree that the targeted courses were more effective for developing problem solving courses than traditional courses

CONCLUSIONS

The Project Catalyst team has developed a conceptual framework for progressively developing students' problem solving skills across the curriculum. The framework consists of three distinct levels of learning outcomes. Work has begun on developing generic curriculum modules that are not course or discipline specific which faculty can use to promote student attainment of the outcomes specified in each framework. Preliminary assessment efforts indicate that both faculty and students perceive that the course sequences in which the problem solving framework was implemented improved students' problem solving skills and that courses were more effective than traditional courses in developing these skills.

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Article #3

Students Use Graphic Organizers to Improve Mathematical Problem-Solving Communications

***This We Believe Characteristics**
• Active Learning • Multiple Learning Approaches • Varied Assessments
*Denotes the corresponding characteristics from NMSA's position paper, *This We Believe*, for this article.

Alan Zollman

"Let me give you a math story problem." This sentence often strikes fear in many middle grades students as well as some teachers. As international comparisons, national commissions, and state assessment results confirm, students have difficulty solving mathematical applications problems (Lester, 2007; U. S. Department of Education Institute of Educational Science, 2007; TIMMS, 2003; McREL, 2002; National Research Council, 2002; Illinois State Board of Education, 1997).

Improving students'problem-solving abilities is a major, if not *the* major, goal of middle grades mathematics (National Council of Teachers of Mathematics, 2000; 1995; 1989). To address this goal, the author, who is a university mathematics educator, and nine inner-city middle school teachers developed a math/science action research project. This article describes our unique approach to mathematical problem solving derived from research on reading and writing pedagogy, specifically, research indicating that students who use graphic organizers to organize their ideas improve their comprehension and communication skills (Goeden, 2002 ; National Reading Panel, 2000).

Many teachers and students use graphic organizers to enhance the writing process in all subject areas, including mathematics. Graphic organizers help students organize and then clarify their thoughts, infer solutions to problems, and communicate their thinking strategies.

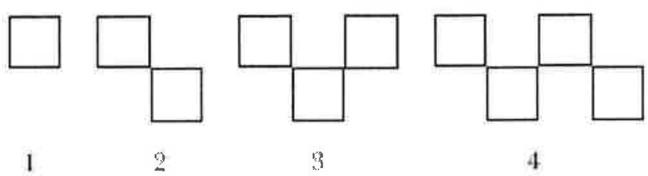
We designed a classroom action research project to study a problem-solving instructional approach in which students used graphic organizers. Our goal was to improve student achievement in three areas of our state's math assessment in open-response problems: mathematics knowledge, strategic knowledge, and mathematical explanation. In this article, we discuss graphic organizers and their potential benefits for both students and teachers, we describe the specific graphic organizer adaptations we created for mathematical problem solving, and we discuss some of our research results of using the *four corners and a diamond* graphic organizer.

Benefits of using graphic organizers in mathematics learning

A graphic organizer is an instructional tool students can use to organize and structure information and concepts and to promote thinking about relationships between concepts. Furthermore, the spatial arrangement of a graphic organizer allows the student, and the teacher, to identify missing information or absent connections in one's strategic thinking (Ellis, 2004).

Middle grades teachers already use many different types of graphic organizers in the writing process. All share the common trait of depicting the process of thinking into a pictorial or graphic format. This helps students reduce and organize information, concepts, and relationships. When a student completes a graphic organizer, he or she does not have to process as much specific, semantic information to understand the information or problem (Ellis, 2004). Graphic organizers allow, and often require, the student to sort information and classify it as essential or non-essential; structure information and concepts; identify relationships between concepts; and organize communication about an issue or problem.

Consider the following middle grades math problem from a recent state assessment.



How many vertices (corners) are there in 1, 2, 3, 4, 5, 6 ... *n* squares when they are arranged in the following way?

What did you first think when reading the problem? Did you first think of the meaning of the term "vertices" or that this is a mathematical pattern problem? Did you first think of counting the corners or that this looks like an arrangement of tables? Did you first think to discuss in your solution why you are not just adding four with every square? Did you first try to think of the singular form of the word vertices?

Initial thinking is not a linear activity, especially in mathematical problem solving. Yet, the result of problem solving—the written solution—often looks like a linear, step-by-step procedure. Good problem solvers brainstorm different thoughts and ideas when first presented with a problem, and these may or may not be useful. Problem solvers can use a graphic organizer to record random information but not process it. A student can later reflect upon usefulness of the information and ideas. If the information and ideas help the student make relationships between concepts, then they are essential. A graphic organizer allows a student to quickly organize, analyze, and synthesize one's knowledge, concepts, relationships, strategy, and communication. It also gives every student a starting point for the problem-solving process.

Adapting a graphic organizer for mathematical problem solving

Figure 1 Four Corners and a diamond mathematics graphic organizer

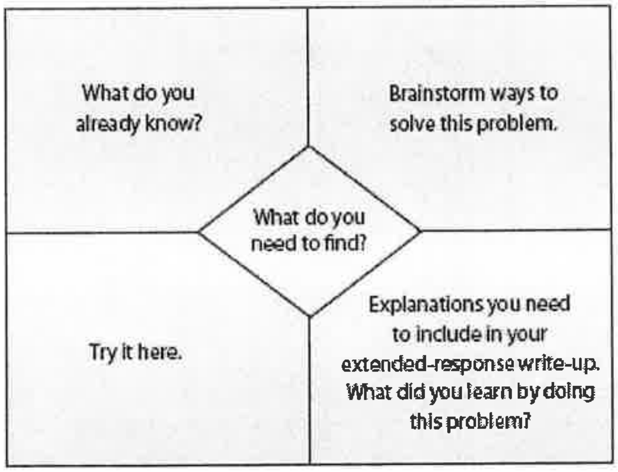


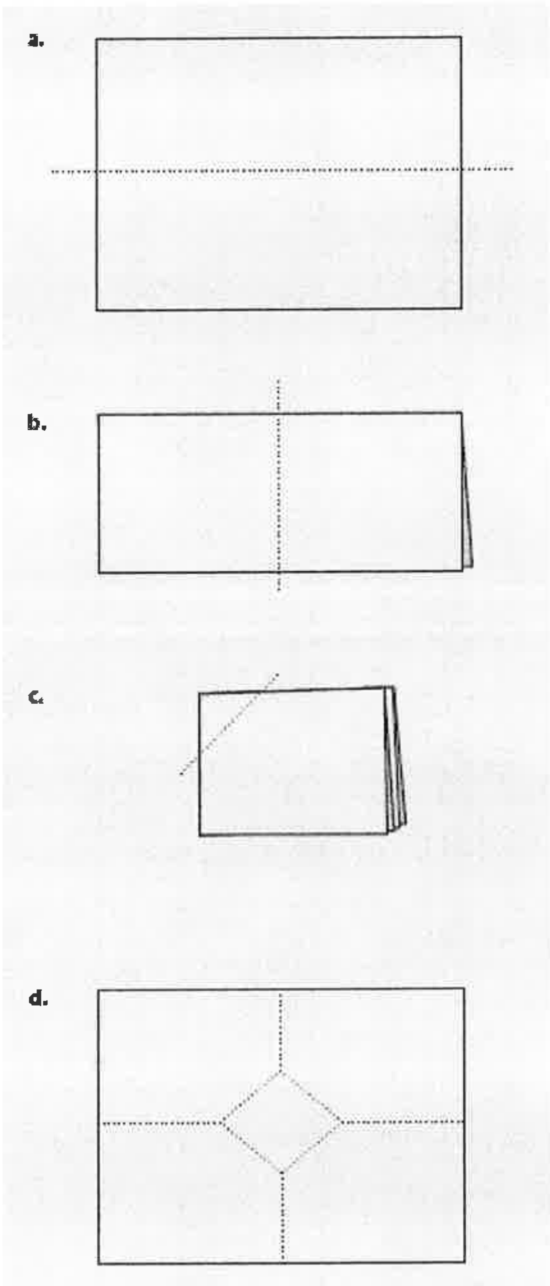
Figure 1 depicts the four corners and a diamond graphic organizer. This graphic organizer was modified from the four squares writing graphic organizer described by Gould and Gould (1999). The four square writing method is a formulaic writing approach, originally designed to teach essay writing to children in a five paragraph, step-by-step approach. The graphic organizer portion of the method specifically assists students with prewriting and organizing. We saw beneficial problem-solving aspects in the graphic organizer portion of this writing method for mathematics.

Our four corners and a diamond graphic organizer has five areas:

1. What do you need to find?
2. What do you already know?
3. Brainstorm possible ways to solve this problem.
4. Try your ways here.
5. What things do you need to include in your response? What mathematics did you learn by working this problem?

Actually, the form in Figure 1 does not have to be given to the students each time. Figure 2 shows how students, using a blank piece of paper, make the four corners and a diamond graphic organizer template. The student folds the paper into fourths, first folding the paper horizontally ("hot dog style"), then vertically ("hamburger style"), and finally the inner corner is folded up. When the paper is unfolded, the creases form the four corners and the "diamond" rhombus in the middle. The teachers reported that students later (e.g., during state testing) often folded or drew the five areas on their paper to begin problem solving.

Figure 2 Four Corners and a diamond folding template



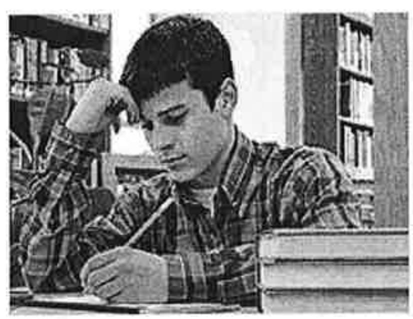
So how does the use of the four corners and a diamond graphic organizer differ from the traditional Polya's four-step mathematical problem-solving hierarchy? In terms of objectives, it does not. Obviously, the four corners and a diamond graphic organizer is designed to help students understand the problem, devise a plan, carry out the plan, and look back (Polya, 1944). However, by having the non-linear layout of the graphic organizer, the student is not expected to do these "steps" in a hierarchical, procedural order that some students misapply. It is the implementation process, how students form their response, that is the important aspect of the four corners and a diamond graphic organizer (Zollman, 2006a).

The pictorial orientation allows students to record their ideas in whatever order they occur. If students first think of the unit for their final answer, then this is recorded in the fifth, bottom-right area. This idea (the unit), then, is not needed in the short-term memory because a reminder is recorded. If students first think of a possible procedure for their answer, this is recorded in the third, upper-right area. The four corners and a diamond graphic organizer allows, and even encourages, students to use their problem-solving strategies in a non-hierarchical order. A student can work in one area of the organizer and later work a different area. It also shows that completing a problem-solving response has several different, but related, aspects.

Students do not begin writing a response until some information or ideas are in all five areas. The four corners and a diamond graphic organizer especially encourages students to begin working on a problem before they have an identified solution method. As in the four square writing method, the students then organize and edit their thoughts by writing their solution in the traditional linear response, using connecting phrases and adding details and relationships. The steps for the open response write-up are as follows: (1) state the problem; (2) list the given information; (3) explain methods for solving the problem; (4) identify mathematical work procedures; and (5) specify the final answer and conclusions.

The graphic portion of the organizer allows all students to fill in parts of the solution process. It encourages all students to persevere—to "muck around" working on a problem. Further, teachers quickly can identify where students are confused when solving a problem by simply examining the graphic organizer.

The teacher should model proper use of the four corners and a diamond graphic organizer and have students work in groups when introducing this tool. Working in groups allows students to see that many problems can be worked in more than one way and that different people start in different places when solving a problem. In their small-group discussions, students identify relationships between the areas in the graphic organizer and among the various solutions.



Graphic organizers can benefit students when they take standardized state mathematics assessments, specifically open-response problem-solving items. Most states use a scoring rubric for these types of items. In Illinois, for example, the scoring rubric has three categories: mathematical knowledge, strategic knowledge, and explanation (Illinois State Board of Education, 2005). Responses are scored on a four-point scale for each category, with scores ranging from zero for "no attempt" to four for "complete." Typically, low-ability students do not attempt to show any work in one or more response categories, while average-ability students often have disorganized responses. Higher-ability students sometimes skip steps in their explanations. The four corners and a diamond graphic organizer helps each type of student produce a more complete response in each of the three categories and, thus, receive a higher score.

Impact of graphic organizers

Nine middle school teachers decided to use the open-response mathematics questions as the focus of their action research on the effects of using graphic organizers. Teachers administered pre- and post-tests with their students to see if using the four corners and a diamond graphic organizer impacted their performance.

All teachers reported dramatic improvements in students' mathematics scores on open-response items after implementing the four corners and a diamond graphic organizer. The percentage of students (N=186) who scored at the "meets" or "exceeds" levels on each of the open-response item categories on the pre-test was 4% for math knowledge, 19% for strategic knowledge, and 8% for explanation. After instructing students to use the graphic organizer in mathematical problem solving, the percentage of students scoring "meets" or "exceeds" on the post-test improved to 75% for math knowledge, 68% for strategic knowledge, and 68% for explanation (Zollman, 2006a; 2006b).

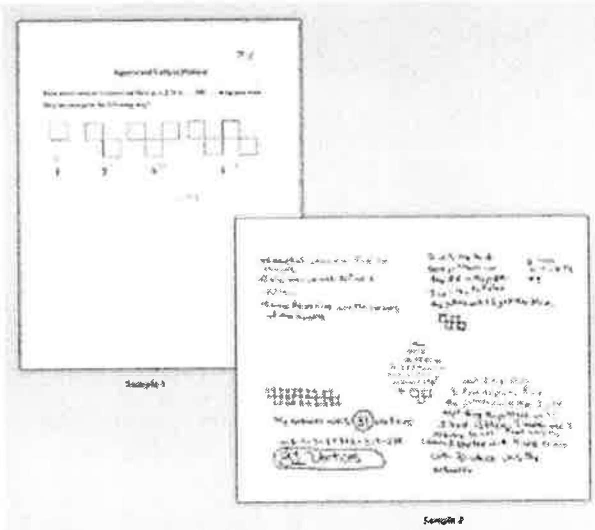
Each teacher self-collected and self-scored these data using the state's scoring rubric. Overall scores increased from a 27% average on the pre-test to a 70% average on the post-test. Data collected, analyzed, and triangulated from three sources—the teachers, the action research pre- and post-test data, and the students' work—suggests that the use of the graphic organizer in mathematical problem solving may significantly help students coordinate their mathematical ideas, methods, thinking, and writing. The graphic organizer helped students coordinate various parts of mathematical problem solving: (a) What is the question? (b) What information is known? (c) What strategies might be used? (d) Which operations, procedures, or algorithms of the strategy need to be shown? (e) What explanations and reflections are needed to communicate the method(s) of solution? (Zollman, 2006a; 2006b).

The teachers found the use of graphic organizers in mathematical problem solving to be very efficient and effective for all levels of students. The teachers saw that their lower-ability students, who normally would not have attempted problems, had now written partial solutions. The organizer appeared to help average-ability students organize thinking strategies and help high-ability students improve their problem-solving communication skills (Zollman, 2006b). Students now had an efficient and familiar method for writing and communicating their thinking in a logical argument.

Samples of students' work

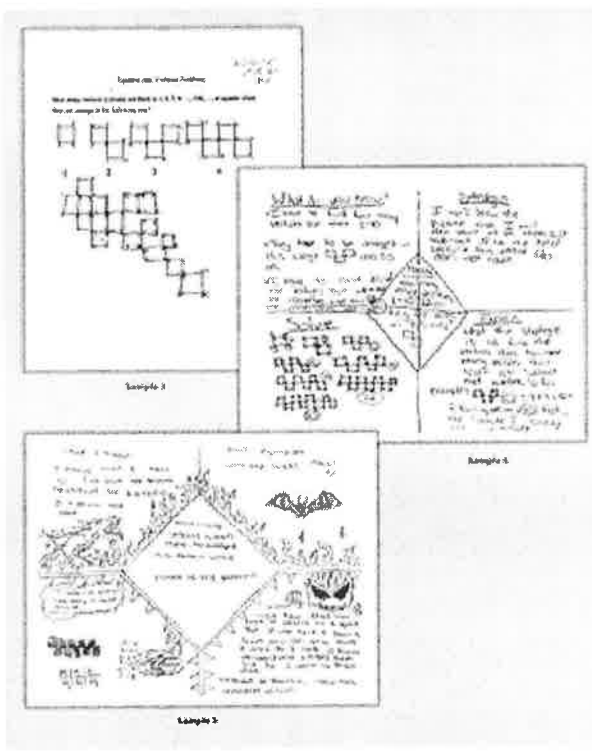
The samples of student work in Figures 3 and 4 are from an open-response squares and vertices problem before and after the use of graphic organizers in the classroom.

Figure 3 Samples 1 & 2 (Click on image of Figure 3 to see larger PDF.)



Sample 1 shows the work of a student who was presented the problem before becoming familiar with the four corners and a diamond graphic organizer. Sample 2 shows the same student's work later in the semester, after learning how to problem solve using the graphic organizer. The student's strategy on the pre-test was to count the individual vertices in the picture, then add these numbers. This work shows a misunderstanding of the problem, limited strategy, and no explanation. On the post-test, this same student's work shows a complete understanding of the problem presented (10 squares) and a complete explanation of a correct strategy that will transfer to other problems, however, it lacks a concluding algebraic formula to demonstrate mathematical knowledge. While it is not a perfect response, understanding, organization, development, and reflection are all strongly represented on the graphic organizer.

Figure 4 Samples 3, 4 & 5 (Click on image of Figure 4 to see larger PDF.)



The second student's pre-test (Sample 3) shows the common incorrect strategy of just counting the total vertices in the picture. It appears that the student then attempted to "add" the individual pictures in the student's own drawing to again count the vertices. However, without any explanation, the teacher cannot know what strategy, if any, the student was attempting. Again, this work shows a misunderstanding of the problem, limited strategy, and no explanation. This student's post-test (Sample 4) illustrates excellent understanding, organization, development and reflection of the problem presented (10 squares). The graphic organizer shows the student's complete, correct strategy, solution, and explanation of the problem. For mathematical knowledge, the formula is well explained in words, not as an algebraic expression. This would be acceptable on state assessments, as the problem did not specifically ask for an algebraic expression.

Sample 5 is the post-test work of a higher-ability student. This student's work demonstrates a full understanding of the problem, a correct solution, and a complete explanation. The drawings also suggest that the student feels a sense of ownership of and satisfaction with the solution and probably finished the problem with plenty of time to spare.

Caveats

We hoped the students in our action research study would improve their problem solving with an instructional intervention from pre-test to post-test; however, no single instructional method directly affects learning. Rather, instruction is one of many factors that may influence learning. Others include the curriculum, the student, the class, and the teacher. Nevertheless, the teachers who conducted the action research described in this article believed the graphic organizer was associated with many of the positive outcomes in their students'problem-solving ability (Zollman, 2006b).

The crucial factor in the effectiveness of any instructional method is how it is implemented. If four corners and a diamond graphic organizer is used as a linear, systematic procedure to teach problem solving, it will succeed sporadically. In fact, any direct teaching about problem solving is likely to have intermittent success. Giving students a chart of Polya's (1944) four steps in problem solving or a graphic organizer sheet may help students learn the steps of problem solving. However, students may remain uncertain about where to start a problem, confused by essential versus non-essential information, or unaware how to communicate important steps and reflections in their solutions. We found that graphic organizers aid students in all three of these areas.

Allowing students to first use their own thinking–and then reflect, revise, and re-organize their knowledge, strategies, and communication–helps them improve their problem-solving abilities. Initially, teaching about problem solving as a hierarchy of procedural steps is neither efficient nor effective. Our results confirm other studies that found teaching *via* problem solving is the key instructional process (Lester, 2007).

Summary

As our work suggests, effective reading and writing strategies like graphic organizers may have crossover effects in mathematics for students of all ability levels. We found that four corners and a diamond, when properly used, was an extremely useful instructional method in the middle grades mathematics classroom. Our instructional approach helped students construct content knowledge and strategic knowledge and, we contend, it also improved their mathematical communication skills. In addition, four corners and a diamond allowed teachers to quickly identify the weaknesses and strengths of students' problem solving abilities. As teachers seek to expand and improve students'mathematical knowledge to help them solve problems, they may find that good teaching in reading and writing is good teaching in math.

Extensions
The author shows how graphic organizers that are typically used to help students organize their thoughts while writing in ELA can also be used to help them think through problem-

solving tasks in mathematics.

How can teachers use graphic organizers to actively engage students in thinking and problem-solving activities in all areas of the curriculum?

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Article #4

Habits of an Effective Problem Solver

One of the instructional goals of the Audio Help files is to assist students in becoming better and more confident problem-solvers. If all students who are good problem-solvers could be observed doing problems, then one would not necessarily observe that they use the same approaches to solving problems. Most good problem-solvers have unique little practices which make them different from other good problem-solvers. Nonetheless, there are several habits which they all share in common. While a good problem-solver may not religiously adhere to these habitual practices, they become more reliant upon them as the problems become more difficult.

The list below describes some of the habits which good problem-solvers share in common. The list is NOT an exhaustive list; it simply includes some commonly observed habits which good problem-solvers practice. Anyone can be a good problem-solver; because of personality and learning style differences, some will certainly be better than others. Nonetheless, anyone who puts effort into disciplining themselves to be successful at solving problems can learn how to be proficient at the task. A student who devotes some time and attention to the list below and makes an effort to personalize it into their own approach to problems will improve their problem-solving ability. The use of these Audio Help files and the problem-solving practices which they promote will not only assist you in completing your problem sets but will also make you a better and more confident problem-solver.

Reading and Visualizing

All good problem-solvers will read a problem carefully and make an effort to visualize the physical situation. Physics problems begin as word problems and terminate as mathematical exercises. Before the mathematics portion of a problem begins, a student must translate the written information into mathematical variables. Many errors (and perhaps even most) can be traced back to this translation process. These errors are usually the result of a failure to visualize the physical situation described in the verbal statement of the problem or of a failure in missing some strategic information during the reading process. A good problem-solver will often construct a diagram of some form to assist in this critical visualization task. The actual diagram will depend upon the topic which the problem pertains to. If the topic pertains to forces, a force diagram might be drawn. If the problem pertains to mirrors, a ray diagram or object-image diagram may be drawn. And if the problem pertains to vector addition, a vector addition diagram may be drawn. But regardless of the topic, a good problem-solver typically begins the translation of the written words into mathematical variables by an informative sketch or diagram which depicts the situation.

Organization of Known and Unknown Information

As mentioned earlier, physics problems begin as word problems and terminate as mathematical exercises. During the algebraic/mathematical part of the problem, the student must make substitution of known numerical information into a mathematical formula (and hopefully into the correct formula). The mathematical formula is written in the form of symbols which represent some physical quantity such as focal length, distance, acceleration or force. Before performing such substitutions, the student must first equate the numerical information contained in the verbal statement with the appropriate physical quantity. It is the habit of a good problem-solver to carefully read the verbal statement and to combine the attention to units (meters, kilograms, Joules, etc.) with their understanding of the meaning of physical quantities in order to accurately extract the numerical information and equate it with the appropriate symbol. Furthermore, good problem-solvers will conduct this task by writing down the quantitative information with its unit and symbol in an organized fashion, often recording the values on their diagram. This task will also include observing strategic and meaningful phrases such as "a magnified and virtual image", "a diverging lens", "starting from rest", "with a constant velocity", and "in the absence of air resistance." While such phrases do not explicitly provide numerical information, they do go a long way towards offering information which implies a particular solution strategy. In addition to identifying the known information, good problem-solvers also practice the habit of identifying the quantity to be solved for, recording it in terms of its appropriate symbol.

Plotting a Strategy for Solving for the Unknown

Once the physical situation has been visualized and diagrammed and the numerical information has been extracted from the verbal statement, the strategy plotting stage begins. During this stage of a problem, the student ponders the question: "How can I use the known information - both explicit and implied - to determine the unknown quantity?" More than any other stage during the problem solution, it is during this stage that a student must think critically and apply their physics knowledge.

Difficult problems in physics (the kind which likely draw students to these audio help files) are multistep problems. The path from known information to the unknown quantity is often not immediately obvious. The problem becomes like a jigsaw puzzle; the assembly of all the pieces into the whole can only occur after careful inspection, thought, analysis, and perhaps some wrong turns. In such cases, the time taken to plot out a strategy will pay huge dividends, preventing the loss of several frustrating minutes of impulsive attempts at solving the problem. Good problem solvers use their background knowledge of physics and physics formulae to think about how the known information is related to each other and how it is related to the final unknown quantity. They know through practice and through observation of other expert problem-solvers (such as their teacher) that there are likely some intermediate unknown quantities which will have to be calculated before finding the final unknown quantity. By comparing the known information (which they have previously written down in an organized manner) to known mathematical formulae, they are able to determine the intermediate quantities which will allow them to subsequently determine the final quantity. They record their thoughts as they think through possible steps for solving the problem; they often sketch a schematic plan that depicts how to put the individual pieces together to solve the problem as a whole.

Often times, difficulties arising in the strategy plotting phase of a problem solution is the result of the lack of knowledge about the topic. A good problem-solver understands that if they know very little about the topic, there is no sense in attempting the problem. Rather than waste valuable time trying, they spend their time learning about the topic, looking for relevant mathematical formulae and studying pertinent concepts and principles. Good problem-solvers are resourceful enough to know where to look to find the formulae and other information which they need to know to solve the problem. They may look in their notes from class, in their instructional packets, in their textbooks or at online resources. Once a good problem-solver has filled their minds with information, they return to the problem to apply their new physics knowledge, asking once more "How can I use the known information - both explicit and implied - to determine the unknown quantity?"

Even with suitable understanding of the physics behind a problem, a student can still get stuck and become in need of help. Good problem-solvers are not typically caught off guard by such sticking points; they understand them to be natural to any strategy plotting process. In such instances, good problem-solvers will often take the time to look at previously done problems which are similar or identical to the one that they are trying to solve. They will compare the current problem to previous ones in terms of known and unknown quantities and observe the solution process to these similar problems, pondering if a similar strategy could be used. They may look at previous problems which they have done, sample problems from the textbook or from online resources, or problems done in class. Because they have taken careful notes from class and organized their own solutions to problems, good problem-solvers benefit tremendously from such comparisons. Often times, the current problem can use the same solution as a previous one. Often times, the mere practice of looking through previous solutions triggers a thought about how one can proceed with the current solution. Considerable learning occurs during this comparison process which allows a good problem-solver to not only solve the current problem but also internalize the mathematical relationships between quantities in physics. This effort makes good problem-solvers into even better problem-solvers, confident to approach any problem that subsequently arises.

Identification of Appropriate Formula(e)

Once a strategy has been plotted for solving a problem, a good problem-solver will list appropriate mathematical formulae on their paper. They may take the time to rearrange the formulae such that the unknown quantity appears by itself on the left side of the equation. They will take the time to inspect the units in which the given information was stated and make conversions to standard metric units if necessary. The process of identifying formula is simply the natural outcome of an effective strategy-plotting phase.

Algebraic Manipulations and Operations

Finally the mathematics begins, but only after the all-important thinking and physics has occurred. In the final step of the solution process, known information is substituted into the identified formulae in order to solve for the unknown quantity. Following the carefully plotted strategy, the good problem-solver takes the time to manipulate the equations and solve for the unknown. They record strategic algebra steps on paper in the event that their answer is wrong. If wrong, they can quickly inspect their algebra to determine if the error occurred during the mathematical phase of the problem or during the planning/thought/physics stage of the problem.

It should be observed in the above description of the habits of a good problem-solver that the majority of work on a problem is done prior to the actual mathematical operations are performed. Physics problems are more than exercises in mathematical manipulation of numerical data. Physics problems require careful reading, good visualization skills, some background physics knowledge, analytical thought and inspection and a lot of strategy-plotting.

Article #5

Promoting Problem-Solving Skills in Elementary Mathematics

Problem solving is an essential, if sometimes neglected, skill that demands attention from the earliest grades.

Problem solving is an essential, if sometimes neglected, skill that demands attention from the earliest grades. Students must learn to question and apply mathematical concepts to problem-solving situations on a regular basis. To support students in this goal, teachers need to

- create a classroom environment that embraces discourse
- bridge the gap between students' ordinary language and the formal language of mathematics
- focus on teaching strategies and conceptual understanding

Promote Discourse

Essential techniques for promoting discourse include *modeling* and *think-alouds*. Through the use of think-alouds, teachers model each stage of the problem-solving process, which can be best understood as following four steps:

1. understanding the question
2. selecting a strategy
3. applying the strategy
4. checking your answer

It is essential to talk through each step of the process. While it is easy to forget to verbalize some of your thinking, keep in mind that students may have little context to understand why a step is taken. Keep your language simple, but be sure your think-aloud is thorough. Use visuals and manipulatives to demonstrate processes when applicable,

The following activities can be used to support this process.

- **Problem of the Day** Give students a problem daily. Instead of solving the problem, break down the task. This makes it easier to model all steps in the problem-solving process. Students can
  - tell what the question is asking them to do
  - underline key words in the question that indicate the mathematical operation to be performed
  - delete extraneous information
  - identify the parts in the question
  - find the best problem-solving strategy and explain why it is the best
  - describe two different ways a problem could have been solved
  - have students develop questions from graphic information
  - share student-generated questions
  - ask other students to solve the problem and justify their answers
- **Sharing Solutions** Choose two to four students who have chosen alternate paths to a problem to share their work. Or, chose one student to share his or her work, and then ask if anyone solved the problem in a different way.
- **Summarizing and Paraphrasing** As each student shares his or her solution, teachers should summarize or paraphrase. This allows the teacher to distill, clarify, and illuminate ideas that students have presented, as well as to link everyday language and mathematical terms to the process.

Focus on Language

Many teachers use word walls to help students communicate their thinking and to understand and use mathematical language. The word wall should be composed of three types of language:

- ***Symbolic***: This refers to mathematical notation, such as +, -, =, %.
- ***Content-specific:*** These are technical words associated with abstract mathematical concepts and skills, such as *sum, addend, product, denominator.*
- ***Academic:*** These terms includes test and discourse language, such as *determine, simplify, predict,* as well as words students use to describe a concept or activity, such as *add, take away.*

The following activities can be used to encourage students to use mathematical language and to support their understanding of the three different types.

- **Word Sort** Make or brainstorm a list of words, associated with one or two related concepts, i.e. addition and subtraction. Write these terms on cards and together with students categorize them. Numerous sorts could be done, i.e. by concept (subtraction/addition); by similar properties, by ordinary language and mathematic language (take away/subtraction). This activity could also be done independently at a math center.
- **Guess My Word Game** Play "guess my word." Give students a clue and have them guess the word (i.e. you use this word when you separate sets). This activity could be adapted for independent math center work. To adapt it for independent learning, make a list of sentence clues and have students work individually or with a partner to try to find the correct word wall term.
- **In Your Own Words** Give students a number of terms from the word wall and have them write a definition, or explain the words meaning in their own words. Research indicates that when students use their own language to define words, they are more likely to retain them.
- **Write a Word Problem** Have the students write problems using the appropriate terminology. Problems can be given to other students to be solved.

Use Whole Group and Small Group Teaching Strategies

Many teachers use a mix of small and whole group instruction, as well as heterogeneous and homogenous groupings to promote strategic thinking. Consider incorporating the following strategies and activities to encourage problem-solving skills.

Whole Group Learning

The most common problem-solving strategies should be modeled initially for the whole group. As strategies are taught or discussed, make a list and keep it on a chart that can be easily referred to. A list could include:

- acting it out
    - drawing a picture
    - making a table
    - looking for a pattern
    - making a list
    - making a model
    - breaking the problem down into smaller parts

Then students share their solutions ask other students to identify what strategy they used.

Small Group and Independent Strategies

- **Use heterogeneous groups.** For cooperative work, pair weaker students with stronger students for selected activities. Research has shown that this grouping benefits both partners, not just the weaker partner. Stronger students solidify their understandings as they verbalize their understanding and try to communicate it to their peer.
- **Offer options to homogenous groups**. Students benefit from being in same- or similar-ability groups as well. Provide options for skill practice that include leveled activities or problems. Students can self-select an activity appropriate to their level. Consider creating a color-coded "math menu" of leveled activities focused on the skills and concepts introduced with direct instruction in the whole group.
- **Include teacher-led and student-led groups.** A three-day process that rotates all students at least once through the teacher-led group and provides small group practice could include the following:
    - Day 1: Assign students to work independently from the math menu. The teacher will work with a target group and after instruction will assign follow-up work.

    - Day 2: The teacher works with a new target group—one group of students works at the 'teacher follow up center' and the third group works from the math menu.

    - Day 3: The teacher works with the last group of students and a three day/3 group rotating system has been established.

Read more about it…

Math Forum

http://mathforum.org/ hosts six different problems of the week projects featuring non-routine word problems for elementary school students. The site comes complete with student mentors who reply to students. This site could be used as a center also.